

1章 数と式

Readiness check ● レディネス チェック

教科書 P.6

問1 (1) $15 \div (-5) \times 3 - 2 \times 3^2$
 $= 15 \div (-5) \times 3 - 2 \times 9$
 $= (-3) \times 3 - 2 \times 9 = -9 - 18 = -27$

(2) $6 - \{7 - (-2) \times (-1)\} \div (-5)$
 $= 6 - (7 - 2) \div (-5) = 6 - 5 \div (-5)$
 $= 6 + 1 = 7$

(3) $\frac{-2^2}{3} - \left(\frac{-2}{3}\right)^2$
 $= \frac{-2 \times 2}{3} - \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$
 $= -\frac{4}{3} - \frac{4}{9} = -\frac{12+4}{9} = -\frac{16}{9}$

問2 (1) $9a - 15 - 2(3a - 5) = 9a - 15 - (6a - 10)$
 $= 9a - 15 - 6a + 10 = 3a - 5$

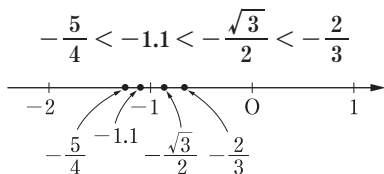
(2) $\frac{2ab - 6a^2b}{2ab} = \frac{2ab(1 - 3a)}{2ab} = 1 - 3a$

(3) $\frac{2x+4}{3} - (5x-3) = \frac{2x+4-3(5x-3)}{3}$
 $= \frac{2x+4-15x+9}{3} = \frac{-13x+13}{3}$

教科書 P.7

問3 $-x^2 + 4x + 5$
 $= -\left(-\frac{2}{3}\right)^2 + 4 \times \left(-\frac{2}{3}\right) + 5$
 $= -\frac{4}{9} - \frac{8}{3} + 5$
 $= \frac{-4 - 24 + 45}{9}$
 $= \frac{17}{9}$

問4 $\sqrt{3} \doteq 1.73$ より $-\frac{\sqrt{3}}{2} \doteq -0.865$
 また $-\frac{5}{4} = -1.25$, $-\frac{2}{3} \doteq -0.667$ であるから



問5 (1) $0.3x - 3 = 0.5(x - 2)$
 両辺に 10 を掛けて
 $3x - 30 = 5(x - 2)$

$$3x - 30 = 5x - 10$$

$$3x - 5x = -10 + 30$$

$$-2x = 20$$

$$x = -10$$

(2) $\frac{2x+4}{3} - (5x-3) = 0$
 両辺に 3 を掛けて
 $2x + 4 - 3(5x - 3) = 0$
 $2x + 4 - 15x + 9 = 0$
 $-13x + 13 = 0$
 $-13x = -13$
 $x = 1$

1節 式の計算

① 整式

教科書 P.8

問1 (1) 次数は 2, 係数は 4
 (2) 次数は 1, 係数は $\frac{1}{3}$
 (3) 次数は 5, 係数は $\frac{3}{2}$
 (4) 次数は 3, 係数は -1

問2 $2x^3 - x^2 + 5x - 3 = 2x^3 + (-x^2) + 5x + (-3)$
 であるから, $2x^3 - x^2 + 5x - 3$ の項は
 $2x^3, -x^2, 5x, -3$

教科書 P.9

問3 (1) $x + 5x^2 - 2 + 7x^3 - 4x$
 $= 7x^3 + 5x^2 + (1 - 4)x - 2$
 $= 7x^3 + 5x^2 - 3x - 2$

(2) $5x - x^2 + 3x^3 + 6x^2 + 3 - 2x^3$
 $= (3 - 2)x^3 + (-1 + 6)x^2 + 5x + 3$
 $= x^3 + 5x^2 + 5x + 3$

問4 (1) 次数の最も高い項は $3x^4$ であるから
 4 次式で, 定数項は -1
 (2) 次数の最も高い項は $-7x^3$ であるから
 3 次式で, 定数項は 2

問5 (1) $x^2 + ax + a^2 - x - 1$
 $= x^2 + (a - 1)x + (a^2 - 1)$
 x については 2 次式で, 定数項は $a^2 - 1$

(2) $x^2 + 2xy - 3y^2 - 3x - 5y + 2$
 $= x^2 + (2y - 3)x + (-3y^2 - 5y + 2)$

x については

2次式で、定数項は $-3y^2 - 5y + 2$

2 整式の加法・減法・乗法

教科書 P.10

問6 (1) $A + B$

$$\begin{aligned} &= (4x^2 - 3x + 10) + (x^2 + x + 6) \\ &= 4x^2 - 3x + 10 + x^2 + x + 6 \\ &= (4 + 1)x^2 + (-3 + 1)x + 10 + 6 \\ &= 5x^2 - 2x + 16 \end{aligned}$$

$A - B$

$$\begin{aligned} &= (4x^2 - 3x + 10) - (x^2 + x + 6) \\ &= 4x^2 - 3x + 10 - x^2 - x - 6 \\ &= (4 - 1)x^2 + (-3 - 1)x + 10 - 6 \\ &= 3x^2 - 4x + 4 \end{aligned}$$

(2) $A + B$

$$\begin{aligned} &= (x^3 - x^2 + 1) + (x^2 + x - 1) \\ &= x^3 - x^2 + 1 + x^2 + x - 1 \\ &= x^3 + (-1 + 1)x^2 + x + 1 - 1 \\ &= x^3 + x \end{aligned}$$

$A - B$

$$\begin{aligned} &= (x^3 - x^2 + 1) - (x^2 + x - 1) \\ &= x^3 - x^2 + 1 - x^2 - x + 1 \\ &= x^3 + (-1 - 1)x^2 - x + 1 + 1 \\ &= x^3 - 2x^2 - x + 2 \end{aligned}$$

問7 (1) $A + 2B$

$$\begin{aligned} &= (3x^2 - 2x + 5) + 2(2x^2 - 4x - 1) \\ &= 3x^2 - 2x + 5 + 4x^2 - 8x - 2 \\ &= (3 + 4)x^2 + (-2 - 8)x + 5 - 2 \\ &= 7x^2 - 10x + 3 \end{aligned}$$

(2) $2A - 3B$

$$\begin{aligned} &= 2(3x^2 - 2x + 5) - 3(2x^2 - 4x - 1) \\ &= 6x^2 - 4x + 10 - 6x^2 + 12x + 3 \\ &= (6 - 6)x^2 + (-4 + 12)x + 10 + 3 \\ &= 8x + 13 \end{aligned}$$

教科書 P.11

問8 (1) $a^6 \times a^3 = a^{6+3} = a^9$

(2) $a \times a^7 = a^{1+7} = a^8$

(3) $(a^5)^3 = a^{5 \times 3} = a^{15}$

(4) $(a^4)^8 = a^{4 \times 8} = a^{32}$

(5) $(ab^4)^2 = a^2(b^4)^2 = a^2b^{4 \times 2} = a^2b^8$

(6) $(a^3b^5)^6 = (a^3)^6(b^5)^6 = a^{3 \times 6}b^{5 \times 6} = a^{18}b^{30}$

問9 (1) $2x^3 \times 3x^5 = (2 \times 3) \times (x^3 \times x^5) = 6x^8$

$$\begin{aligned} (2) \quad &9xy \times (-5x^4) \\ &= \{9 \times (-5)\} \times (xy \times x^4) \\ &= \{9 \times (-5)\} \times \{(x \times x^4) \times y\} \\ &= -45x^5y \end{aligned}$$

$$\begin{aligned} (3) \quad &(3x^3)^4 \times 10x^2 \\ &= 3^4(x^3)^4 \times 10x^2 \\ &= (3^4 \times 10) \times \{(x^3)^4 \times x^2\} \\ &= (81 \times 10) \times (x^{12} \times x^2) \\ &= 810x^{14} \end{aligned}$$

$$\begin{aligned} (4) \quad &(-2xy^3)^2 \times (3xy)^3 \\ &= (-2)^2x^2(y^3)^2 \times 3^3x^3y^3 \\ &= \{(-2)^2 \times 3^3\} \times (x^2 \times x^3) \times \{(y^3)^2 \times y^3\} \\ &= (4 \times 27) \times x^5 \times (y^6 \times y^3) \\ &= 108x^5y^9 \end{aligned}$$

教科書 P.12

問10 (1) $4x(x^2 + 4x - 3)$

$$\begin{aligned} &= 4x \cdot x^2 + 4x \cdot 4x + 4x \cdot (-3) \\ &= 4x^3 + 16x^2 - 12x \end{aligned}$$

$$\begin{aligned} (2) \quad &(3x^2 - 2x + 5) \times (-2x) \\ &= 3x^2 \cdot (-2x) - 2x \cdot (-2x) + 5 \cdot (-2x) \\ &= -6x^3 + 4x^2 - 10x \end{aligned}$$

問11 (1) $(x + 6)(2x + 3)$

$$\begin{aligned} &= x(2x + 3) + 6(2x + 3) \\ &= 2x^2 + 3x + 12x + 18 \\ &= 2x^2 + (3 + 12)x + 18 \\ &= 2x^2 + 15x + 18 \end{aligned}$$

$$\begin{aligned} (2) \quad &(3x - 2)(x - 1) \\ &= 3x(x - 1) - 2(x - 1) \\ &= 3x^2 - 3x - 2x + 2 \\ &= 3x^2 + (-3 - 2)x + 2 \\ &= 3x^2 - 5x + 2 \end{aligned}$$

$$\begin{aligned} (3) \quad &(x + 5)(2x^2 - 3x - 6) \\ &= x(2x^2 - 3x - 6) + 5(2x^2 - 3x - 6) \\ &= 2x^3 - 3x^2 - 6x + 10x^2 - 15x - 30 \\ &= 2x^3 + (-3 + 10)x^2 + (-6 - 15)x - 30 \\ &= 2x^3 + 7x^2 - 21x - 30 \end{aligned}$$

$$\begin{aligned} (4) \quad &(2x - 3)(4x^2 - x + 2) \\ &= 2x(4x^2 - x + 2) - 3(4x^2 - x + 2) \\ &= 8x^3 - 2x^2 + 4x - 12x^2 + 3x - 6 \end{aligned}$$

$$= 8x^3 + (-2 - 12)x^2 + (4 + 3)x - 6$$

$$= 8x^3 - 14x^2 + 7x - 6$$

教科書 P.13

問12 (1) $(x+2)^2 = x^2 + 2 \cdot x \cdot 2 + 2^2$
 $= x^2 + 4x + 4$

(2) $(x-5)^2 = x^2 - 2 \cdot x \cdot 5 + 5^2$
 $= x^2 - 10x + 25$

(3) $(x+3y)^2$
 $= x^2 + 2 \cdot x \cdot 3y + (3y)^2$
 $= x^2 + 6xy + 9y^2$

(4) $(3x-4y)^2$
 $= (3x)^2 - 2 \cdot 3x \cdot 4y + (4y)^2$
 $= 9x^2 - 24xy + 16y^2$

(5) $(3x+2)(3x-2)$
 $= (3x)^2 - 2^2 = 9x^2 - 4$

(6) $(5x+2y)(5x-2y)$
 $= (5x)^2 - (2y)^2 = 25x^2 - 4y^2$

問13 (1) $(x+5)(x+3)$
 $= x^2 + (5+3)x + 5 \cdot 3$
 $= x^2 + 8x + 15$

(2) $(x-3)(x+6)$
 $= x^2 + (-3+6)x + (-3) \cdot 6$
 $= x^2 + 3x - 18$

(3) $(x+4y)(x-7y)$
 $= x^2 + (4y-7y)x + 4y \cdot (-7y)$
 $= x^2 - 3xy - 28y^2$

(4) $(x-y)(x-5y)$
 $= x^2 + (-y-5y)x + (-y) \cdot (-5y)$
 $= x^2 - 6xy + 5y^2$

教科書 P.14

問14 (1) $(3x+4)(2x+3)$
 $= 3 \cdot 2x^2 + (3 \cdot 3 + 4 \cdot 2)x + 4 \cdot 3$
 $= 6x^2 + 17x + 12$

(2) $(4x+1)(5x-2)$
 $= 4 \cdot 5x^2 + \{4 \cdot (-2) + 1 \cdot 5\}x + 1 \cdot (-2)$
 $= 20x^2 - 3x - 2$

(3) $(2x-3y)(x+5y)$
 $= 2 \cdot 1x^2 + \{2 \cdot 5y + (-3y) \cdot 1\}x$
 $\quad \quad \quad + (-3y) \cdot 5y$
 $= 2x^2 + 7xy - 15y^2$

(4) $(3x-2y)(4x-3y)$

$$= 3 \cdot 4x^2 + \{3 \cdot (-3y) + (-2y) \cdot 4\}x$$

$$\quad \quad \quad + (-2y) \cdot (-3y)$$

$$= 12x^2 - 17xy + 6y^2$$

問15 (1) $a-b = A$ とおくと
 $(a-b+3)(a-b-7)$

$$= (A+3)(A-7)$$

$$= A^2 - 4A - 21$$

$$= (a-b)^2 - 4(a-b) - 21$$

$$= a^2 - 2ab + b^2 - 4a + 4b - 21$$

(2) $x+y = A$ とおくと
 $(x+y)(x+y-z)$

$$= A(A-z)$$

$$= A^2 - Az$$

$$= (x+y)^2 - (x+y)z$$

$$= x^2 + 2xy + y^2 - xz - yz$$

教科書 P.15

問16 (1) $(a-b-2)^2$
 $= a^2 + (-b)^2 + (-2)^2 + 2 \cdot a \cdot (-b)$
 $\quad \quad \quad + 2 \cdot (-b) \cdot (-2) + 2 \cdot (-2) \cdot a$
 $= a^2 + b^2 + 4 - 2ab + 4b - 4a$

(2) $(a-3b+2c)^2$
 $= a^2 + (-3b)^2 + (2c)^2 + 2 \cdot a \cdot (-3b)$
 $\quad \quad \quad + 2 \cdot (-3b) \cdot 2c + 2 \cdot 2c \cdot a$
 $= a^2 + 9b^2 + 4c^2 - 6ab - 12bc + 4ca$

発展

3次式の乗法公式

教科書 P.16

問1 [1] $(a+b)^3 = (a+b)(a+b)^2$
 $= (a+b)(a^2 + 2ab + b^2)$
 $= a \cdot a^2 + a \cdot 2ab + a \cdot b^2 + b \cdot a^2$
 $\quad \quad \quad + b \cdot 2ab + b \cdot b^2$
 $= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
 $= a^3 + 3a^2b + 3ab^2 + b^3$

[2] $(a-b)^3 = (a-b)(a-b)^2$
 $= (a-b)(a^2 - 2ab + b^2)$
 $= a \cdot a^2 + a \cdot (-2ab) + a \cdot b^2 - b \cdot a^2$
 $\quad \quad \quad - b \cdot (-2ab) - b \cdot b^2$
 $= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$
 $= a^3 - 3a^2b + 3ab^2 - b^3$

問2 (1) $(x+1)^3 = x^3 + 3 \cdot x^2 \cdot 1 + 3 \cdot x \cdot 1^2 + 1^3$
 $= x^3 + 3x^2 + 3x + 1$

(2) $(2x-3)^3$

$$= (2x)^3 - 3 \cdot (2x)^2 \cdot 3 + 3 \cdot 2x \cdot 3^2 - 3^3$$

$$= 8x^3 - 36x^2 + 54x - 27$$

$$(3) (3x + y)^3$$

$$= (3x)^3 + 3 \cdot (3x)^2 \cdot y + 3 \cdot 3x \cdot y^2 + y^3$$

$$= 27x^3 + 27x^2y + 9xy^2 + y^3$$

$$(4) (x - 2y)^3$$

$$= x^3 - 3 \cdot x^2 \cdot 2y + 3 \cdot x \cdot (2y)^2 - (2y)^3$$

$$= x^3 - 6x^2y + 12xy^2 - 8y^3$$

問3 [3] $(a + b)(a^2 - ab + b^2)$

$$= a \cdot a^2 + a \cdot (-ab) + a \cdot b^2 + b \cdot a^2$$

$$+ b \cdot (-ab) + b \cdot b^2$$

$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$

$$= a^3 + b^3$$

[4] $(a - b)(a^2 + ab + b^2)$

$$= a \cdot a^2 + a \cdot ab + a \cdot b^2 - b \cdot a^2$$

$$- b \cdot ab - b \cdot b^2$$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

問4 (1) $(x + 5)(x^2 - 5x + 25)$

$$= (x + 5)(x^2 - 5 \cdot x + 5^2)$$

$$= x^3 + 5^3 = x^3 + 125$$

(2) $(4x - 3y)(16x^2 + 12xy + 9y^2)$

$$= (4x - 3y)\{(4x)^2 + 4x \cdot 3y + (3y)^2\}$$

$$= (4x)^3 - (3y)^3 = 64x^3 - 27y^3$$

3 因數分解

教科書 P.17

問17 (1) $xy + xz = x(y + z)$

(2) $3a^2b + b = (3a^2 + 1)b$

(3) $abc - acd = ac \cdot b - ac \cdot d$

$$= ac(b - d)$$

(4) $12x^2y + 18xy^2 = 6xy \cdot 2x + 6xy \cdot 3y$

$$= 6xy(2x + 3y)$$

教科書 P.18

問18 (1) $x^2 + 4x + 4 = x^2 + 2 \cdot 2 \cdot x + 2^2$

$$= (x + 2)^2$$

(2) $4x^2 - 20xy + 25y^2$

$$= (2x)^2 - 2 \cdot 2x \cdot 5y + (5y)^2$$

$$= (2x - 5y)^2$$

(3) $9x^2 - 25 = (3x)^2 - 5^2$

$$= (3x + 5)(3x - 5)$$

(4) $36x^2 - 49y^2 = (6x)^2 - (7y)^2$

$$= (6x + 7y)(6x - 7y)$$

問19 (1) $x^2 + 5x + 6$

$$= x^2 + (2 + 3)x + 2 \cdot 3$$

$$= (x + 2)(x + 3)$$

(2) $x^2 - x - 12$

$$= x^2 + \{3 + (-4)\}x + 3 \cdot (-4)$$

$$= (x + 3)(x - 4)$$

(3) $x^2 - 9x + 18$

$$= x^2 + \{(-3) + (-6)\}x + (-3) \cdot (-6)$$

$$= (x - 3)(x - 6)$$

(4) $x^2 + 5x - 24$

$$= x^2 + \{(-3) + 8\}x + (-3) \cdot 8$$

$$= (x - 3)(x + 8)$$

教科書 P.19

問20 (1) $x^2 + 6xy + 8y^2$

$$= x^2 + 6y \cdot x + 8y^2$$

$$= (x + 2y)(x + 4y) \quad \left\{ \begin{array}{l} 8y^2 = 2y \cdot 4y \\ 6y = 2y + 4y \end{array} \right.$$

(2) $x^2 - 3xy - 18y^2$

$$= x^2 - 3y \cdot x - 18y^2$$

$$= (x - 6y)(x + 3y) \quad \left\{ \begin{array}{l} -18y^2 = (-6y) \cdot 3y \\ -3y = (-6y) + 3y \end{array} \right.$$

教科書 P.20

問21 (1) $2x^2 + 3x + 1$

$$= (x + 1)(2x + 1) \quad \begin{array}{r} 1 \times 1 \longrightarrow 2 \\ 2 \times 1 \longrightarrow 1 \\ \hline 3 \end{array}$$

(2) $5x^2 - 12x + 4$

$$= (x - 2)(5x - 2) \quad \begin{array}{r} 1 \times -2 \longrightarrow -10 \\ 5 \times -2 \longrightarrow -2 \\ \hline -12 \end{array}$$

(3) $8x^2 + 2x - 3$

$$= (2x - 1)(4x + 3) \quad \begin{array}{r} 2 \times -1 \longrightarrow -4 \\ 4 \times 3 \longrightarrow 6 \\ \hline 2 \end{array}$$

(4) $4x^2 - 11x + 6$

$$= (x - 2)(4x - 3) \quad \begin{array}{r} 1 \times -2 \longrightarrow -8 \\ 4 \times -3 \longrightarrow -12 \\ \hline -11 \end{array}$$

(5) $12x^2 - x - 6$

$$= (3x + 2)(4x - 3) \quad \begin{array}{r} 3 \times 2 \longrightarrow 8 \\ 4 \times -3 \longrightarrow -12 \\ \hline -1 \end{array}$$

(6) $6x^2 - 13x + 6$

$$= (2x - 3)(3x - 2) \quad \begin{array}{r} 2 \times -3 \longrightarrow -9 \\ 3 \times -2 \longrightarrow -6 \\ \hline -13 \end{array}$$

問22 (1) $4x^2 + 3xy - 7y^2$ $\begin{array}{r} 1 \times -y \rightarrow -4y \\ 4 \times 7y \rightarrow 7y \\ \hline 3y \end{array}$
 $= 4x^2 + 3y \cdot x - 7y^2$
 $= (x - y)(4x + 7y)$

(2) $8x^2 - 2xy - 15y^2$
 $= 8x^2 - 2y \cdot x - 15y^2$
 $= (2x - 3y)(4x + 5y)$

$$\begin{array}{r} 2 \times -3y \rightarrow -12y \\ 4 \times 5y \rightarrow 10y \\ \hline -2y \end{array}$$

教科書 P.21

問23 (1) $x + y = A$ とおくと
 $x(x + y) + 5y(x + y)$
 $= xA + 5yA$
 $= A(x + 5y)$
 $= (x + y)(x + 5y)$

(2) $a - b = A$ とおくと
 $(a - b)^2 - 3(a - b)$
 $= A^2 - 3A = A(A - 3)$
 $= (a - b)(a - b - 3)$

(3) $a - b = A$ とおくと
 $x(a - b) + b - a$
 $= x(a - b) - (a - b)$
 $= xA - A$
 $= A(x - 1)$
 $= (a - b)(x - 1)$

問24 (1) $x + y = A$ とおくと
 $(x + y)^2 + 7(x + y) + 10$
 $= A^2 + 7A + 10$
 $= (A + 2)(A + 5)$
 $= (x + y + 2)(x + y + 5)$

(2) $x + 2y = A$ とおくと
 $(x + 2y)^2 - 6(x + 2y) + 9$
 $= A^2 - 6A + 9$
 $= (A - 3)^2$
 $= (x + 2y - 3)^2$

(3) $y + z = A$ とおくと
 $x^2 - (y + z)^2$
 $= x^2 - A^2$
 $= (x + A)(x - A)$
 $= \{x + (y + z)\}\{x - (y + z)\}$
 $= (x + y + z)(x - y - z)$

問25 (1) y について整理すると
 $x^2 + xy - x + y - 2$
 $= y(x + 1) + (x^2 - x - 2)$
 $= y(x + 1) + (x + 1)(x - 2)$
 $= (x + 1)\{y + (x - 2)\}$
 $= (x + 1)(x + y - 2)$

(2) a について整理すると
 $2ab + 2b^2 - a + b - 1$
 $= a(2b - 1) + (2b^2 + b - 1)$
 $= a(2b - 1) + (2b - 1)(b + 1)$
 $= (2b - 1)(a + b + 1)$

教科書 P.22

問26 (1) x について整理すると
 $x^2 + 4xy + 3y^2 - 4x - 14y - 5$
 $= x^2 + (4y - 4)x + (3y^2 - 14y - 5)$
 $= x^2 + (4y - 4)x + (y - 5)(3y + 1)$
 $= \{x + (y - 5)\}\{x + (3y + 1)\}$
 $= (x + y - 5)(x + 3y + 1)$

$$\begin{array}{r} y \times -5 \rightarrow -15y \\ 3y \times 1 \rightarrow 3y \\ \hline -14y \end{array}$$

(2) x について整理すると
 $3x^2 + 2xy - y^2 - x + 3y - 2$
 $= 3x^2 + (2y - 1)x - (y^2 - 3y + 2)$
 $= 3x^2 + (2y - 1)x - (y - 1)(y - 2)$
 $= \{x + (y - 1)\}\{3x - (y - 2)\}$
 $= (x + y - 1)(3x - y + 2)$

$$\begin{array}{r} 1 \times y - 1 \rightarrow 3y - 3 \\ 3 \times -(y - 2) \rightarrow -y + 2 \\ \hline 2y - 1 \end{array}$$

発展 3次式の因数分解

問1 (1) $x^3 + 64 = x^3 + 4^3$
 $= (x + 4)(x^2 - x \cdot 4 + 4^2)$
 $= (x + 4)(x^2 - 4x + 16)$

(2) $x^3 - 1 = x^3 - 1^3$
 $= (x - 1)(x^2 + x \cdot 1 + 1^2)$
 $= (x - 1)(x^2 + x + 1)$

(3) $27x^3 + y^3 = (3x)^3 + y^3$
 $= (3x + y)\{(3x)^2 - 3x \cdot y + y^2\}$
 $= (3x + y)(9x^2 - 3xy + y^2)$

1 (1) $A - B - C$
 $= (x^2 + x - 3) - (2x^2 - x + 4) - (-3x^2 + 5)$
 $= x^2 + x - 3 - 2x^2 + x - 4 + 3x^2 - 5$
 $= (1 - 2 + 3)x^2 + (1 + 1)x + (-3 - 4 - 5)$
 $= 2x^2 + 2x - 12$

(2) $3(2A + B) - 2(3A - C)$
 $= 6A + 3B - 6A + 2C = 3B + 2C$
 $= 3(2x^2 - x + 4) + 2(-3x^2 + 5)$
 $= 6x^2 - 3x + 12 - 6x^2 + 10$
 $= -3x + 22$

2 (1) $4a^5 \times 3a^2 = 4 \cdot 3 \cdot a^{5+2} = 12a^7$

(2) $-x^3 \times (-x)^4$
 $= -x^3 \cdot x^4 = -x^{3+4} = -x^7$

(3) $5a^3b \times (-7a^4b^5)$
 $= 5 \cdot (-7) \cdot a^{3+4} \cdot b^{1+5}$
 $= -35a^7b^6$

(4) $(-2xy)^3 \times (3x^2y^3)^2$
 $= (-2)^3 x^3 y^3 \times 3^2 (x^2)^2 (y^3)^2$
 $= \{(-2)^3 \cdot 3^2\} \times \{x^3 \cdot (x^2)^2\} \times \{y^3 \cdot (y^3)^2\}$
 $= -72x^7y^9$

3 (1) $5xy(x^2 - xy + 3y^2)$
 $= 5xy \cdot x^2 + 5xy(-xy) + 5xy \cdot 3y^2$
 $= 5x^3y - 5x^2y^2 + 15xy^3$

(2) $(3x - 1)(x^2 + 7x + 5)$
 $= 3x(x^2 + 7x + 5) - (x^2 + 7x + 5)$
 $= 3x^3 + 21x^2 + 15x - x^2 - 7x - 5$
 $= 3x^3 + 20x^2 + 8x - 5$

(3) $(9x + 2y)^2$
 $= (9x)^2 + 2 \cdot 9x \cdot 2y + (2y)^2$
 $= 81x^2 + 36xy + 4y^2$

(4) $(6x - 7y)^2$
 $= (6x)^2 - 2 \cdot 6x \cdot 7y + (7y)^2$
 $= 36x^2 - 84xy + 49y^2$

(5) $(3x + 10y)(3x - 10y)$
 $= (3x)^2 - (10y)^2$
 $= 9x^2 - 100y^2$

(6) $(x - 8y)(x + 6y)$
 $= x^2 + (-8y + 6y)x - 8y \cdot 6y$
 $= x^2 - 2xy - 48y^2$

(7) $(5x - 2y)(3x - y)$
 $= 5 \cdot 3x^2 + \{5 \cdot (-y) - 2y \cdot 3\}x - 2y \cdot (-y)$
 $= 15x^2 - 11xy + 2y^2$

(8) $(4x + 5y)(5x - 4y)$
 $= 4 \cdot 5x^2 + \{4 \cdot (-4y) + 5y \cdot 5\}x + 5y \cdot (-4y)$
 $= 20x^2 + 9xy - 20y^2$

4 (1) $a + c = A$ とおくと
 $(a + b + c)(a - b + c)$
 $= (A + b)(A - b)$
 $= A^2 - b^2$
 $= (a + c)^2 - b^2$
 $= a^2 + 2ac + c^2 - b^2$
 $= a^2 - b^2 + c^2 + 2ac$

(2) $(2a - 3b + 1)^2$
 $= (2a)^2 + (-3b)^2 + 1^2 + 2 \cdot 2a \cdot (-3b)$
 $\quad + 2 \cdot (-3b) \cdot 1 + 2 \cdot 1 \cdot 2a$
 $= 4a^2 + 9b^2 + 1 - 12ab - 6b + 4a$
 $= 4a^2 - 12ab + 9b^2 + 4a - 6b + 1$

5 (1) $3a^3b^2 - 6a^2b^3 + 12a^2b^2c$
 $= 3a^2b^2 \cdot a - 3a^2b^2 \cdot 2b + 3a^2b^2 \cdot 4c$
 $= 3a^2b^2(a - 2b + 4c)$

(2) $x^2 - 8x + 16 = x^2 - 2 \cdot 4 \cdot x + 4^2$
 $= (x - 4)^2$

(3) $16a^2 + 24ab + 9b^2$
 $= (4a)^2 + 2 \cdot 4a \cdot 3b + (3b)^2$
 $= (4a + 3b)^2$

(4) $16x^2 - 81y^2 = (4x)^2 - (9y)^2$
 $= (4x + 9y)(4x - 9y)$

(5) $x^2 - 11x + 10$
 $= x^2 + (-1 - 10)x + (-1) \cdot (-10)$
 $= (x - 1)(x - 10)$

(6) $x^2 + 3xy - 54y^2$
 $= x^2 + 3y \cdot x - 54y^2$
 $= x^2 + \{(-6y) + 9y\}x + (-6y) \cdot 9y$
 $= (x + 9y)(x - 6y)$

(7) $10x^2 + 17x + 6$ $\begin{array}{r} 2 \times 1 \longrightarrow 5 \\ 5 \times 6 \longrightarrow 12 \\ \hline 17 \end{array}$
 $= (2x + 1)(5x + 6)$

(8) $8x^2 - 13x - 6$ $\begin{array}{r} 1 \times -2 \longrightarrow -16 \\ 8 \times 3 \longrightarrow 3 \\ \hline -13 \end{array}$
 $= (x - 2)(8x + 3)$

$$\begin{aligned} (9) \quad & 15x^2 - 22xy + 8y^2 \\ &= 15x^2 - 22y \cdot x + 8y^2 \\ &= (3x - 2y)(5x - 4y) \end{aligned}$$

$$\begin{array}{r} 3 \quad \times \quad -2y \longrightarrow -10y \\ 5 \quad \times \quad -4y \longrightarrow -12y \\ \hline -22y \end{array}$$

$$\begin{aligned} (10) \quad & 6x^2 + 23xy - 18y^2 \\ &= 6x^2 + 23y \cdot x - 18y^2 \\ &= (2x + 9y)(3x - 2y) \end{aligned}$$

$$\begin{array}{r} 2 \quad \times \quad 9y \longrightarrow 27y \\ 3 \quad \times \quad -2y \longrightarrow -4y \\ \hline 23y \end{array}$$

$$\begin{aligned} 6 \quad (1) \quad & 2x^3 - 12x^2 + 18x \\ &= 2x(x^2 - 6x + 9) \\ &= 2x(x - 3)^2 \end{aligned}$$

$$\begin{aligned} (2) \quad & ax^2 - 9ay^2 \\ &= a(x^2 - 9y^2) \\ &= a\{x^2 - (3y)^2\} \\ &= a(x + 3y)(x - 3y) \end{aligned}$$

$$\begin{aligned} (3) \quad & x - 3y = A \quad \text{とおくと} \\ & x(x - 3y) - 4y(3y - x) \\ &= x(x - 3y) + 4y(x - 3y) \\ &= xA + 4yA \\ &= (x + 4y)A \\ &= (x + 4y)(x - 3y) \end{aligned}$$

$$\begin{aligned} (4) \quad & 2x + y = A \quad \text{とおくと} \\ & (2x + y)^2 + 6(2x + y) - 7 \\ &= A^2 + 6A - 7 \\ &= (A + 7)(A - 1) \\ &= (2x + y + 7)(2x + y - 1) \end{aligned}$$

$$\begin{aligned} (5) \quad & x - y = A \quad \text{とおくと} \\ & 2(x - y)^2 + (y - x) - 3 \\ &= 2(x - y)^2 - (x - y) - 3 \\ &= 2A^2 - A - 3 \\ &= (A + 1)(2A - 3) \\ &= (x - y + 1)(2x - 2y - 3) \end{aligned}$$

$$\begin{array}{r} 1 \quad \times \quad 1 \longrightarrow 2 \\ 2 \quad \times \quad -3 \longrightarrow -3 \\ \hline -1 \end{array}$$

$$\begin{aligned} (6) \quad & b \text{ について整理すると} \\ & a^2b - 3ab + a + 2b - 2 \\ &= (a^2 - 3a + 2)b + (a - 2) \\ &= (a - 2)(a - 1)b + (a - 2) \\ &= (a - 2)\{(a - 1)b + 1\} \\ &= (a - 2)(ab - b + 1) \end{aligned}$$

$$\begin{aligned} (7) \quad & x \text{ について整理すると} \\ & 2x^2 + 5xy + 2y^2 - 5x - y - 3 \\ &= 2x^2 + (5y - 5)x + (2y^2 - y - 3) \\ &= 2x^2 + (5y - 5)x + (y + 1)(2y - 3) \\ &= \{x + (2y - 3)\}\{2x + (y + 1)\} \\ &= (x + 2y - 3)(2x + y + 1) \end{aligned}$$

$$\begin{array}{r} 1 \quad \times \quad 1 \longrightarrow 2 \\ 2 \quad \times \quad -3 \longrightarrow -3 \\ \hline -1 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad 2y - 3 \longrightarrow 4y - 6 \\ 2 \quad \times \quad y + 1 \longrightarrow y + 1 \\ \hline 5y - 5 \end{array}$$

$$\begin{aligned} (8) \quad & x \text{ について整理すると} \\ & x^2 - y^2 + 4x + 6y - 5 \\ &= x^2 + 4x - (y^2 - 6y + 5) \\ &= x^2 + 4x - (y - 1)(y - 5) \\ &= \{x + (y - 1)\}\{x - (y - 5)\} \\ &= (x + y - 1)(x - y + 5) \end{aligned}$$

2 節 実数

① 実数

教科書 P.24

問 1 (1) $0.3 = \frac{3}{10}$

(2) $2.04 = \frac{204}{100} = \frac{51}{25}$

(3) $0.025 = \frac{25}{1000} = \frac{1}{40}$

教科書 P.25

問 2 (1) $\frac{5}{6} = 0.8333\cdots = 0.8\dot{3}$

(2) $\frac{3}{11} = 0.272727\cdots = 0.2\dot{7}$

(3) $\frac{7}{27} = 0.259259259\cdots = 0.2\dot{5}9$

参考 循環小数を分数で表す

問1 $x = 0.\dot{2}7$ とおくと, $x = 0.272727\cdots$ であるから

$$100x = 27.2727\cdots$$

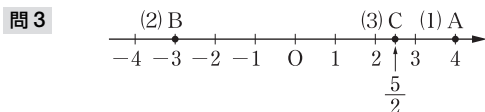
よって

$$\begin{array}{r} 100x = 27.2727\cdots \\ -) \quad x = 0.2727\cdots \\ \hline 99x = 27 \end{array}$$

$$x = \frac{27}{99} = \frac{3}{11}$$

したがって $0.\dot{2}7 = \frac{3}{11}$

教科書 P.27



教科書 P.28

問4 (1) $|4| = 4$ (2) $|-5| = 5$

(3) $|0| = 0$

問5 $x = 7$ と $x = -7$

問6 (1) $|2-7| = |-5| = 5$

(2) $\left|\frac{1}{3} - \frac{1}{4}\right| = \left|\frac{1}{12}\right| = \frac{1}{12}$

(3) $\sqrt{2} < 2$ であるから

$$|\sqrt{2} - 2| = -(\sqrt{2} - 2) = 2 - \sqrt{2}$$

2 根号を含む式の計算

教科書 P.29

問7 (1) $\sqrt{7}, -\sqrt{7}$

(2) $9, -9$

(3) $\frac{1}{2}, -\frac{1}{2}$

教科書 P.30

問8 (1) $\sqrt{3}\sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$

(2) $\frac{\sqrt{42}}{\sqrt{6}} = \sqrt{\frac{42}{6}} = \sqrt{7}$

問9 (1) $\sqrt{8} = \sqrt{2^2 \times 2} = \sqrt{2^2} \times \sqrt{2} = 2\sqrt{2}$

(2) $\sqrt{24} = \sqrt{2^2 \times 6} = \sqrt{2^2} \times \sqrt{6} = 2\sqrt{6}$

(3) $\sqrt{50} = \sqrt{5^2 \times 2} = \sqrt{5^2} \times \sqrt{2} = 5\sqrt{2}$

(4) $\sqrt{\frac{32}{9}} = \frac{\sqrt{32}}{\sqrt{9}} = \frac{\sqrt{4^2 \times 2}}{\sqrt{3^2}} = \frac{\sqrt{4^2} \times \sqrt{2}}{\sqrt{3^2}}$

$$= \frac{4\sqrt{2}}{3}$$

(5) $\sqrt{\frac{27}{16}} = \frac{\sqrt{27}}{\sqrt{16}} = \frac{\sqrt{3^2 \times 3}}{\sqrt{4^2}}$

$$= \frac{\sqrt{3^2} \times \sqrt{3}}{\sqrt{4^2}} = \frac{3\sqrt{3}}{4}$$

問10 (1) $\sqrt{6} \times \sqrt{8} = \sqrt{6 \times 8} = \sqrt{2 \times 3 \times 2^3}$

$$= \sqrt{2^4 \times 3} = \sqrt{4^2 \times 3} = 4\sqrt{3}$$

(2) $\sqrt{20} \times \sqrt{10} = \sqrt{20 \times 10} = \sqrt{2 \times 10 \times 10}$

$$= \sqrt{10^2 \times 2} = 10\sqrt{2}$$

(3) $\sqrt{15} \times \sqrt{21} = \sqrt{15 \times 21} = \sqrt{3 \times 5 \times 3 \times 7}$

$$= \sqrt{3^2 \times 5 \times 7} = 3\sqrt{35}$$

(4) $\sqrt{12} \times \sqrt{8} \times \sqrt{24} = 2\sqrt{3} \times 2\sqrt{2} \times 2\sqrt{6}$

$$= 8\sqrt{3 \times 2 \times 6} = 8\sqrt{6^2} = 8 \cdot 6 = 48$$

教科書 P.31

問11 (1) $\sqrt{7} - 3\sqrt{7} + 4\sqrt{7} = (1-3+4)\sqrt{7}$

$$= 2\sqrt{7}$$

(2) $\sqrt{48} + 2\sqrt{3} = \sqrt{4^2 \times 3} + 2\sqrt{3}$

$$= 4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$$

(3) $\sqrt{20} + \sqrt{\frac{5}{9}} = \sqrt{2^2 \times 5} + \frac{\sqrt{5}}{\sqrt{3^2}}$

$$= 2\sqrt{5} + \frac{\sqrt{5}}{3}$$

$$= \left(2 + \frac{1}{3}\right)\sqrt{5} = \frac{7\sqrt{5}}{3}$$

(4) $\sqrt{\frac{3}{16}} - \sqrt{\frac{12}{25}} = \frac{\sqrt{3}}{\sqrt{4^2}} - \frac{\sqrt{2^2 \times 3}}{\sqrt{5^2}}$

$$= \frac{\sqrt{3}}{4} - \frac{2\sqrt{3}}{5}$$

$$= \left(\frac{1}{4} - \frac{2}{5}\right)\sqrt{3}$$

$$= -\frac{3\sqrt{3}}{20}$$

(5) $\sqrt{72} - \sqrt{75} + \sqrt{108} - \sqrt{128}$

$$= \sqrt{6^2 \times 2} - \sqrt{5^2 \times 3} + \sqrt{6^2 \times 3} - \sqrt{8^2 \times 2}$$

$$= 6\sqrt{2} - 5\sqrt{3} + 6\sqrt{3} - 8\sqrt{2}$$

$$= \sqrt{3} - 2\sqrt{2}$$

問12 (1) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

$$= (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$$

(2) $(2\sqrt{2} + 3)^2$

$$= (2\sqrt{2})^2 + 2 \times 2\sqrt{2} \times 3 + 3^2$$

$$= 8 + 12\sqrt{2} + 9$$

$$= 17 + 12\sqrt{2}$$

(3) $(\sqrt{15} - \sqrt{6})^2$

$$= (\sqrt{15})^2 - 2 \times \sqrt{15} \times \sqrt{6} + (\sqrt{6})^2$$

$$= 15 - 2\sqrt{15 \times 6} + 6$$

$$= 21 - 2\sqrt{3^2 \times 10}$$

$$= 21 - 6\sqrt{10}$$

$$(4) (3\sqrt{2} + \sqrt{3})(\sqrt{2} + 2\sqrt{3})$$

$$= 3\sqrt{2} \times \sqrt{2} + 3\sqrt{2} \times 2\sqrt{3} + \sqrt{3} \times \sqrt{2} + \sqrt{3} \times 2\sqrt{3}$$

$$= 6 + 6\sqrt{6} + \sqrt{6} + 6$$

$$= 12 + 7\sqrt{6}$$

教科書 P.32

$$\text{問13 (1)} \quad \frac{1}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$(2) \quad \frac{6}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$(3) \quad \frac{1}{\sqrt{18}} = \frac{1}{3\sqrt{2}} = \frac{1 \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{6}$$

$$(4) \quad \frac{6}{\sqrt{24}} = \frac{6}{2\sqrt{6}} = \frac{6 \times \sqrt{6}}{2\sqrt{6} \times \sqrt{6}} \\ = \frac{6 \times \sqrt{6}}{2 \times 6} = \frac{\sqrt{6}}{2}$$

$$\text{問14 (1)} \quad \frac{1}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\ = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ = \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \\ = \sqrt{3} - \sqrt{2}$$

$$(2) \quad \frac{5}{\sqrt{7} - \sqrt{2}} = \frac{5(\sqrt{7} + \sqrt{2})}{(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})} \\ = \frac{5(\sqrt{7} + \sqrt{2})}{(\sqrt{7})^2 - (\sqrt{2})^2} \\ = \frac{5(\sqrt{7} + \sqrt{2})}{7 - 2} \\ = \frac{5(\sqrt{7} + \sqrt{2})}{5} \\ = \sqrt{7} + \sqrt{2}$$

$$(3) \quad \frac{\sqrt{7} + 3}{\sqrt{7} - 3} = \frac{(\sqrt{7} + 3)^2}{(\sqrt{7} - 3)(\sqrt{7} + 3)} \\ = \frac{(\sqrt{7})^2 + 2 \times \sqrt{7} \times 3 + 3^2}{(\sqrt{7})^2 - 3^2} \\ = \frac{7 + 6\sqrt{7} + 9}{7 - 9} \\ = \frac{16 + 6\sqrt{7}}{-2}$$

$$= -8 - 3\sqrt{7}$$

教科書 P.33

$$\text{問15} \quad x = \frac{\sqrt{7} + \sqrt{5}}{(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})} = \frac{\sqrt{7} + \sqrt{5}}{2}$$

$$y = \frac{\sqrt{7} - \sqrt{5}}{(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})} = \frac{\sqrt{7} - \sqrt{5}}{2}$$

$$(1) \quad x + y = \frac{\sqrt{7} + \sqrt{5}}{2} + \frac{\sqrt{7} - \sqrt{5}}{2} = \sqrt{7}$$

$$(2) \quad xy = \frac{\sqrt{7} + \sqrt{5}}{2} \times \frac{\sqrt{7} - \sqrt{5}}{2} \\ = \frac{(\sqrt{7})^2 - (\sqrt{5})^2}{2^2}$$

$$= \frac{7 - 5}{4} = \frac{1}{2}$$

$$(3) \quad x^2 + y^2 = (x + y)^2 - 2xy \\ = (\sqrt{7})^2 - 2 \times \frac{1}{2} \\ = 7 - 1 \\ = 6$$

Training トレーニング

$$7 (1) \quad |a| + |b| = |5| + |-8| \\ = 5 + 8 = 13$$

$$(2) \quad |a + b| = |5 + (-8)| \\ = |-3| = 3$$

$$(3) \quad |a| - |b| = |5| - |-8| \\ = 5 - 8 = -3$$

$$(4) \quad |a - b| = |5 - (-8)| \\ = |13| = 13$$

$$8 (1) \quad \sqrt{13} \times \sqrt{65} \times \sqrt{15} = \sqrt{13 \times 65 \times 15} \\ = \sqrt{13 \times 13 \times 5 \times 3 \times 5} = \sqrt{13^2 \times 5^2 \times 3} \\ = 13 \times 5 \times \sqrt{3} \\ = 65\sqrt{3}$$

$$(2) \quad \sqrt{12} + \sqrt{48} - \sqrt{27} \\ = \sqrt{2^2 \times 3} + \sqrt{4^2 \times 3} - \sqrt{3^2 \times 3} \\ = 2\sqrt{3} + 4\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$$

$$(3) \quad (\sqrt{7} + \sqrt{6})^2 \\ = (\sqrt{7})^2 + 2 \times \sqrt{7} \times \sqrt{6} + (\sqrt{6})^2 \\ = 7 + 2\sqrt{42} + 6 = 13 + 2\sqrt{42}$$

$$(4) \quad (2\sqrt{3} - \sqrt{5})^2 \\ = (2\sqrt{3})^2 - 2 \times 2\sqrt{3} \times \sqrt{5} + (\sqrt{5})^2 \\ = 12 - 4\sqrt{15} + 5$$

$$= 17 - 4\sqrt{15}$$

$$(5) (\sqrt{2} - 3)(3\sqrt{2} + 1)$$

$$= \sqrt{2} \times 3\sqrt{2} + \sqrt{2} \times 1 - 3 \times 3\sqrt{2} - 3 \times 1$$

$$= 6 + \sqrt{2} - 9\sqrt{2} - 3$$

$$= 3 - 8\sqrt{2}$$

$$(6) 8(4 - \sqrt{7}) - (4 - \sqrt{7})^2$$

$$= (4 - \sqrt{7})\{8 - (4 - \sqrt{7})\}$$

$$= (4 - \sqrt{7})(4 + \sqrt{7})$$

$$= 4^2 - (\sqrt{7})^2$$

$$= 16 - 7$$

$$= 9$$

$$9 (1) \frac{6}{\sqrt{75}} = \frac{6}{\sqrt{5^2 \times 3}} = \frac{6}{5\sqrt{3}}$$

$$= \frac{6 \times \sqrt{3}}{5\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{15}$$

$$= \frac{2\sqrt{3}}{5}$$

$$(2) \frac{4\sqrt{5}}{\sqrt{54}} = \frac{4\sqrt{5}}{\sqrt{3^2 \times 6}} = \frac{4\sqrt{5}}{3\sqrt{6}}$$

$$= \frac{4\sqrt{5} \times \sqrt{6}}{3\sqrt{6} \times \sqrt{6}} = \frac{4\sqrt{30}}{18}$$

$$= \frac{2\sqrt{30}}{9}$$

$$(3) \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \frac{(\sqrt{5} + 1)^2}{(\sqrt{5} - 1)(\sqrt{5} + 1)}$$

$$= \frac{(\sqrt{5})^2 + 2 \times \sqrt{5} \times 1 + 1^2}{(\sqrt{5})^2 - 1^2}$$

$$= \frac{5 + 2\sqrt{5} + 1}{5 - 1} = \frac{6 + 2\sqrt{5}}{4}$$

$$= \frac{3 + \sqrt{5}}{2}$$

$$(4) \frac{\sqrt{2} + 2\sqrt{3}}{2\sqrt{2} + \sqrt{3}}$$

$$= \frac{(\sqrt{2} + 2\sqrt{3})(2\sqrt{2} - \sqrt{3})}{(2\sqrt{2} + \sqrt{3})(2\sqrt{2} - \sqrt{3})}$$

$$= \frac{\sqrt{2} \times 2\sqrt{2} - \sqrt{2} \times \sqrt{3} + 2\sqrt{3} \times 2\sqrt{2} - 2\sqrt{3} \times \sqrt{3}}{(2\sqrt{2})^2 - (\sqrt{3})^2}$$

$$= \frac{4 - \sqrt{6} + 4\sqrt{6} - 6}{8 - 3}$$

$$= \frac{-2 + 3\sqrt{6}}{5}$$

$$10 \quad x = \frac{\sqrt{5} - 2}{(\sqrt{5} + 2)(\sqrt{5} - 2)} = \sqrt{5} - 2$$

$$y = \frac{\sqrt{5} + 2}{(\sqrt{5} - 2)(\sqrt{5} + 2)} = \sqrt{5} + 2$$

$$x + y = (\sqrt{5} - 2) + (\sqrt{5} + 2) = 2\sqrt{5}$$

$$xy = (\sqrt{5} - 2)(\sqrt{5} + 2) = (\sqrt{5})^2 - 2^2 = 1$$

$$(1) \quad x^2 + xy + y^2 = (x + y)^2 - xy$$

$$= (2\sqrt{5})^2 - 1$$

$$= 20 - 1 = 19$$

$$(2) \quad \frac{y}{x} + \frac{x}{y} = \frac{y^2 + x^2}{xy} = \frac{(x + y)^2 - 2xy}{xy}$$

$$= \frac{(2\sqrt{5})^2 - 2 \times 1}{1}$$

$$= 20 - 2 = 18$$

Challenge チャレンジ 例題 整数部分と小数部分

教科書 P.34

$$\text{問 1 (1)} \quad \frac{1}{\sqrt{5} - 2} = \frac{\sqrt{5} + 2}{(\sqrt{5} - 2)(\sqrt{5} + 2)}$$

$$= \frac{\sqrt{5} + 2}{5 - 4} = \sqrt{5} + 2$$

$$2^2 < 5 < 3^2 \text{ より } 2 < \sqrt{5} < 3$$

よって $\sqrt{5}$ の整数部分は 2

したがって、 $\frac{1}{\sqrt{5} - 2} = \sqrt{5} + 2$ の整数部

分 a は $a = 2 + 2 = 4$

また、小数部分 b は

$$b = \frac{1}{\sqrt{5} - 2} - a = (\sqrt{5} + 2) - 4 = \sqrt{5} - 2$$

すなわち $a = 4$, $b = \sqrt{5} - 2$

$$(2) \quad \frac{3}{\sqrt{7} - 2} = \frac{3(\sqrt{7} + 2)}{(\sqrt{7} - 2)(\sqrt{7} + 2)}$$

$$= \frac{3(\sqrt{7} + 2)}{7 - 4} = \frac{3(\sqrt{7} + 2)}{3}$$

$$= \sqrt{7} + 2$$

$$2^2 < 7 < 3^2 \text{ より } 2 < \sqrt{7} < 3$$

よって $\sqrt{7}$ の整数部分は 2

したがって、 $\frac{3}{\sqrt{7} - 2} = \sqrt{7} + 2$ の整数部

分 a は $a = 2 + 2 = 4$

また、小数部分 b は

$$b = \frac{3}{\sqrt{7} - 2} - a = (\sqrt{7} + 2) - 4 = \sqrt{7} - 2$$

すなわち $a = 4$, $b = \sqrt{7} - 2$

教科書 P.35

- 問1 (1) $\sqrt{4+2\sqrt{3}} = \sqrt{(3+1)+2\sqrt{3}\times 1}$
 $= \sqrt{3}+1$
- (2) $\sqrt{6-2\sqrt{8}} = \sqrt{(4+2)-2\sqrt{4}\times 2}$
 $= \sqrt{4}-\sqrt{2} = 2-\sqrt{2}$
- (3) $\sqrt{7+\sqrt{24}} = \sqrt{7+2\sqrt{6}}$
 $= \sqrt{(6+1)+2\sqrt{6}\times 1}$
 $= \sqrt{6}+1$
- (4) $\sqrt{11-4\sqrt{6}} = \sqrt{11-2\sqrt{24}}$
 $= \sqrt{(8+3)-2\sqrt{8}\times 3}$
 $= \sqrt{8}-\sqrt{3} = 2\sqrt{2}-\sqrt{3}$

3 節 1次不等式

1 不等式の性質

教科書 P.36

- 問1 (1) $3x-8 < 10$
 (2) $2a+3b \geq 300$

教科書 P.37

- 問2 (1) $2a = -20, 2b = 4$ より
 $2a < 2b$
 $\frac{a}{2} = -5, \frac{b}{2} = 1$ より
 $\frac{a}{2} < \frac{b}{2}$
- (2) $2a = -12, 2b = -4$ より
 $2a < 2b$
 $\frac{a}{2} = -3, \frac{b}{2} = -1$ より
 $\frac{a}{2} < \frac{b}{2}$

- 問3 (1) $(-2)a = 12, (-2)b = -8$ より
 $(-2)a > (-2)b$
 $\frac{a}{-2} = 3, \frac{b}{-2} = -2$ より
 $\frac{a}{-2} > \frac{b}{-2}$
- (2) $(-2)a = 16, (-2)b = 4$ より
 $(-2)a > (-2)b$
 $\frac{a}{-2} = 4, \frac{b}{-2} = 1$ より
 $\frac{a}{-2} > \frac{b}{-2}$

2 1次不等式

教科書 P.38

- 問4 (1) $x+7 > 4$
 7を右辺に移項すると
 $x > 4-7$
 $x > -3$
- (2) $x-5 \leq -3$
 -5を右辺に移項すると
 $x \leq -3+5$
 $x \leq 2$

教科書 P.39

- 問5 (1) $8x-9 < 7$
 -9を右辺に移項すると
 $8x < 7+9$
 整理すると $8x < 16$
 両辺を8で割ると $x < 2$
- (2) $3x+27 \geq 0$
 27を右辺に移項すると $3x \geq -27$
 両辺を3で割ると $x \geq -9$

- 問6 (1) $9x+4 < 7x-6$
 4を右辺に, $7x$ を左辺に移項すると
 $9x-7x < -6-4$
 整理すると $2x < -10$
 両辺を2で割ると $x < -5$
- (2) $4-9x \geq 1-3x$
 4を右辺に, $-3x$ を左辺に移項すると
 $-9x+3x \geq 1-4$
 整理すると $-6x \geq -3$
 両辺を-6で割ると $x \leq \frac{1}{2}$

教科書 P.40

- 問7 (1) $4(x+1) > x-5$
 $4x+4 > x-5$
 $3x > -9$
 両辺を3で割ると $x > -3$
- (2) $6x-3(2x-5) < 4x+5$
 $6x-6x+15 < 4x+5$
 $-4x < -10$
 両辺を-4で割ると $x > \frac{5}{2}$

- 問8 (1) $\frac{x-1}{4} \leq 2-x$
 不等式の両辺に4を掛けて

$$4 \times \frac{x-1}{4} \leq 4(2-x)$$

$$x-1 \leq 8-4x$$

$$5x \leq 9$$

$$\text{両辺を5で割ると } x \leq \frac{9}{5}$$

$$(2) \quad \frac{x}{2} - \frac{2}{3} \geq \frac{5(x-2)}{6}$$

不等式の両辺に6を掛けて

$$6 \times \frac{x}{2} - 6 \times \frac{2}{3} \geq 6 \times \frac{5(x-2)}{6}$$

$$3x-4 \geq 5(x-2)$$

$$3x-4 \geq 5x-10$$

$$-2x \geq -6$$

$$\text{両辺を}-2\text{で割ると } x \leq 3$$

3 1次不等式の応用

教科書 P.41

問9 1回に運べる荷物を x 個とすると、 x 個の荷物の重さは $50x$ kg であるから、2人で運ぶときの全体の重さは

$$(50x+120) \text{ kg}$$

となる。これが 750 kg 以下であることから

$$50x+120 \leq 750$$

$$50x \leq 630$$

$$\text{よって } x \leq \frac{630}{50} = 12.6$$

これを満たす最大の整数 x は 12 である。

したがって、荷物は 12 個まで運ぶことができる。

教科書 P.42

$$\text{問10 (1) } \begin{cases} 2x-5 < 3 & \dots\dots ① \\ 4x+7 > x-2 & \dots\dots ② \end{cases}$$

$$\text{①より } 2x < 8$$

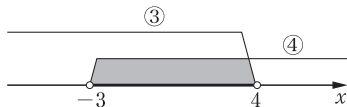
両辺を2で割って

$$x < 4 \quad \dots\dots ③$$

$$\text{②より } 3x > -9$$

両辺を3で割って

$$x > -3 \quad \dots\dots ④$$



求める解は ③, ④ の共通の範囲であるから

$$-3 < x < 4$$

$$(2) \quad \begin{cases} 3x-1 < x+5 & \dots\dots ① \\ x \leq 2(x+1) & \dots\dots ② \end{cases}$$

$$\text{①より } 2x < 6$$

両辺を2で割って

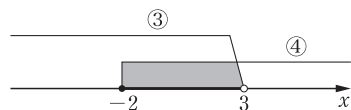
$$x < 3 \quad \dots\dots ③$$

$$\text{②より } x \leq 2x+2$$

$$-x \leq 2$$

両辺を-1で割って

$$x \geq -2 \quad \dots\dots ④$$



求める解は ③, ④ の共通の範囲であるから

$$-2 \leq x < 3$$

教科書 P.43

問11 この不等式を解くには、次の連立不等式を解けばよい。

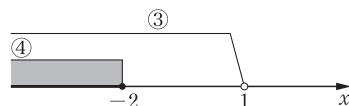
$$\begin{cases} 3x-2 < x & \dots\dots ① \\ x \leq -2x-6 & \dots\dots ② \end{cases}$$

$$\text{①より } 2x < 2 \text{ となるから}$$

$$x < 1 \quad \dots\dots ③$$

$$\text{②より } 3x \leq -6 \text{ となるから}$$

$$x \leq -2 \quad \dots\dots ④$$



求める解は ③, ④ の共通の範囲であるから

$$x \leq -2$$

Training トレーニング

$$\text{11 (1) } 2+5x < 3x-6$$

$$2x < -8$$

$$x < -4$$

$$(2) 7-3x \leq 2x-3$$

$$-5x \leq -10$$

$$x \geq 2$$

$$(3) 2x+7 < 2(3x-4)-5$$

$$2x+7 < 6x-8-5$$

$$-4x < -20$$

$$x > 5$$

$$(4) 4x+6(3-x) \geq 32$$

$$4x+18-6x \geq 32$$

$$\begin{aligned}
 & -2x \geq 14 \\
 & \quad x \leq -7 \\
 (5) \quad & \frac{3-2x}{6} \geq \frac{x+8}{4} - x \\
 & 12 \times \frac{3-2x}{6} \geq 12 \times \frac{x+8}{4} - 12x \\
 & 2(3-2x) \geq 3(x+8) - 12x \\
 & 6-4x \geq 3x+24-12x \\
 & 5x \geq 18 \\
 & \quad x \geq \frac{18}{5}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & 0.4(2x-1) \leq 0.3x+1.6 \\
 & 10 \times 0.4(2x-1) \leq 10(0.3x+1.6) \\
 & 4(2x-1) \leq 3x+16 \\
 & 8x-4 \leq 3x+16 \\
 & 5x \leq 20 \\
 & \quad x \leq 4
 \end{aligned}$$

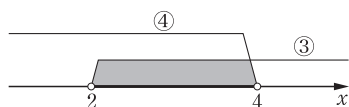
12 ゆりを x 本買うとすると、ばらの本数は $(16-x)$ 本である。代金が 5000 円以下になることから

$$\begin{aligned}
 & 320x + 240(16-x) + 400 \leq 5000 \\
 & \text{両辺を 10 で割って} \\
 & 32x + 24(16-x) + 40 \leq 500 \\
 & 32x + 384 - 24x + 40 \leq 500 \\
 & 8x \leq 76 \\
 & \quad x \leq 9.5
 \end{aligned}$$

これを満たす最大の整数は 9 である。また、ばらの本数は、 $16-9=7$

したがって、**ゆりを 9 本、ばらを 7 本** 買えばよい。

$$\begin{aligned}
 13 \quad (1) \quad & \begin{cases} 6-4x < -2 & \dots\dots ① \\ 2x-8 < 3(4-x) & \dots\dots ② \end{cases} \\
 & \text{① より } -4x < -8 \\
 & \quad x > 2 \quad \dots\dots ③ \\
 & \text{② より } 2x-8 < 12-3x \\
 & \quad 5x < 20 \\
 & \quad x < 4 \quad \dots\dots ④
 \end{aligned}$$

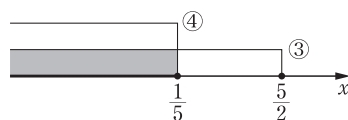


求める解は ③、④ の共通の範囲であるから $2 < x < 4$

$$(2) \quad \begin{cases} x \leq 3(2-x) + 4 & \dots\dots ① \\ \frac{x-1}{2} \leq -\frac{x+1}{3} & \dots\dots ② \end{cases}$$

$$\begin{aligned}
 \text{① より } & x \leq 6-3x+4 \\
 & 4x \leq 10 \\
 & \quad x \leq \frac{5}{2} \quad \dots\dots ③
 \end{aligned}$$

$$\begin{aligned}
 \text{② より } & 6 \times \frac{x-1}{2} \leq 6 \times \left(-\frac{x+1}{3}\right) \\
 & 3(x-1) \leq -2(x+1) \\
 & 3x-3 \leq -2x-2 \\
 & 5x \leq 1 \\
 & \quad x \leq \frac{1}{5} \quad \dots\dots ④
 \end{aligned}$$



求める解は ③、④ の共通の範囲であるから

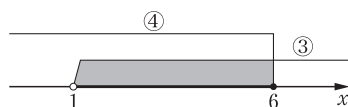
$$x \leq \frac{1}{5}$$

14 この不等式を解くには、次の連立不等式を解けばよい。

$$\begin{cases} 3-2x < 3x-2 & \dots\dots ① \\ 3x-2 \leq 10+x & \dots\dots ② \end{cases}$$

$$\begin{aligned}
 \text{① より } & -5x < -5 \\
 & \quad x > 1 \quad \dots\dots ③
 \end{aligned}$$

$$\begin{aligned}
 \text{② より } & 2x \leq 12 \\
 & \quad x \leq 6 \quad \dots\dots ④
 \end{aligned}$$



求める解は ③、④ の共通の範囲であるから

$$1 < x \leq 6$$

◆ 参考 絶対値記号を含む方程式・不等式

教科書 P.44

問 1 (1) $|x-2| = 4$
 $x-2 = \pm 4$ より $x = 2 \pm 4$
すなわち $x = 6, -2$

(2) $|x+7| = 3$
 $x+7 = \pm 3$ より $x = -7 \pm 3$
すなわち $x = -4, -10$

問 2 (1) $|2x| < 4$
 $-4 < 2x < 4$

各辺を2で割って

$$-2 < x < 2$$

$$(2) |x+2| \leq 5$$

$$-5 \leq x+2 \leq 5$$

各辺から2を引いて

$$-7 \leq x \leq 3$$

教科書 P.45

問3 (1) $|x+5| > 6$

$$x+5 < -6 \quad \text{または} \quad 6 < x+5$$

それぞれの不等式を解いて

$$x < -11, \quad 1 < x$$

$$(2) |3x| \geq 9$$

$$3x \leq -9 \quad \text{または} \quad 9 \leq 3x$$

それぞれの不等式を解いて

$$x \leq -3, \quad 3 \leq x$$

問4 (i) $x-5 \geq 0$ すなわち $x \geq 5$ ……①

のとき, $|x-5| = x-5$ であるから

$$x-5 = 2x-1$$

$$-x = 4$$

よって $x = -4$ ……②

②は①を満たさないから, 解ではない。

(ii) $x-5 < 0$ すなわち $x < 5$ ……③

のとき, $|x-5| = -(x-5)$ であるから

$$-(x-5) = 2x-1$$

$$-x+5 = 2x-1$$

$$-3x = -6$$

よって $x = 2$ ……④

④は③を満たす。

(i), (ii) より, 求める解は $x = 2$

[Level Up]

教科書 P.46

1 $A+B = 6x^2 - 3x - 4$ ……①

$A-B = 4x^2 + 7x + 12$ ……②

①+②より $2A = 10x^2 + 4x + 8$

①-②より $2B = 2x^2 - 10x - 16$

よって $A = 5x^2 + 2x + 4$

$$B = x^2 - 5x - 8$$

2 (1) $(x+1)(x+2)(x+3)(x+4)$

$$= \{(x+1)(x+4)\}\{(x+2)(x+3)\}$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6)$$

$$= \{(x^2 + 5x) + 4\}\{(x^2 + 5x) + 6\}$$

$$x^2 + 5x = A \quad \text{とおくと}$$

$$\{(x^2 + 5x) + 4\}\{(x^2 + 5x) + 6\}$$

$$= (A+4)(A+6)$$

$$= A^2 + 10A + 24$$

$$= (x^2 + 5x)^2 + 10(x^2 + 5x) + 24$$

$$= x^4 + 10x^3 + 25x^2 + 10x^2 + 50x + 24$$

$$= x^4 + 10x^3 + 35x^2 + 50x + 24$$

(2) $(x-2)(x-1)(x+4)(x+8)$

$$= \{(x-2)(x+4)\}\{(x-1)(x+8)\}$$

$$= (x^2 + 2x - 8)(x^2 + 7x - 8)$$

$$= \{(x^2 - 8) + 2x\}\{(x^2 - 8) + 7x\}$$

$$x^2 - 8 = A \quad \text{とおくと}$$

$$\{(x^2 - 8) + 2x\}\{(x^2 - 8) + 7x\}$$

$$= (A+2x)(A+7x)$$

$$= A^2 + (2x+7x)A + 2x \cdot 7x$$

$$= A^2 + 9xA + 14x^2$$

$$= (x^2 - 8)^2 + 9x(x^2 - 8) + 14x^2$$

$$= x^4 - 16x^2 + 64 + 9x^3 - 72x + 14x^2$$

$$= x^4 + 9x^3 - 2x^2 - 72x + 64$$

(3) $(a+b-c+d)(a-b+c+d)$

$$= \{(a+d) + (b-c)\}\{(a+d) - (b-c)\}$$

$$= (a+d)^2 - (b-c)^2$$

$$= a^2 + 2ad + d^2 - (b^2 - 2bc + c^2)$$

$$= a^2 - b^2 - c^2 + d^2 + 2ad + 2bc$$

(4) $(x-2)(x+2)(x^2+4)(x^4+16)$

$$= \{(x-2)(x+2)\}\{(x^2+4)(x^4+16)\}$$

$$= (x^2-4)(x^2+4)(x^4+16)$$

$$= \{(x^2-4)(x^2+4)\}(x^4+16)$$

$$= (x^4-16)(x^4+16)$$

$$= x^8 - 256$$

3 $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

より

$$x^2 + y^2 + z^2 = (x+y+z)^2 - 2(xy + yz + zx)$$

$$= 2^2 - 2 \times 1 = 4 - 2 = 2$$

4 (1) $x^2 - y^2 - z^2 + 2yz$

$$= x^2 - (y^2 - 2yz + z^2)$$

$$= x^2 - (y-z)^2$$

$$= \{x + (y-z)\}\{x - (y-z)\}$$

$$= (x+y-z)(x-y+z)$$

(2) $(x-3)(x-1)(x+2)(x+4) + 24$

$$\begin{aligned}
&= \{(x-3)(x+4)\}(x-1)(x+2) + 24 \\
&= (x^2+x-12)(x^2+x-2) + 24 \\
&\quad x^2+x = A \text{ とおく} \\
&\quad (x^2+x-12)(x^2+x-2) + 24 \\
&= (A-12)(A-2) + 24 \\
&= A^2 - 14A + 24 + 24 \\
&= A^2 - 14A + 48 \\
&= (A-6)(A-8) \\
&= (x^2+x-6)(x^2+x-8) \\
&= (x+3)(x-2)(x^2+x-8)
\end{aligned}$$

$$\begin{aligned}
(3) \quad &x^4 - 7x^2 + 12 \\
&= (x^2)^2 - 7x^2 + 12 \\
&= (x^2-4)(x^2-3) \\
&= (x+2)(x-2)(x^2-3)
\end{aligned}$$

$$\begin{aligned}
(4) \quad &a+1 = A \text{ とおく} \\
&\quad (a+b+c+1)(a+1) + bc \\
&= (A+b+c)A + bc \\
&= A^2 + (b+c)A + bc \\
&= (A+b)(A+c) \\
&= (a+1+b)(a+1+c) \\
&= (a+b+1)(a+c+1)
\end{aligned}$$

$$\begin{aligned}
(5) \quad &a(b^2-c^2) + b(c^2-a^2) + c(a^2-b^2) \\
&= (c-b)a^2 + (b^2-c^2)a + bc^2 - b^2c \\
&= (c-b)a^2 + (b+c)(b-c)a + bc(c-b) \\
&= (c-b)a^2 - (b+c)(c-b)a + bc(c-b) \\
&= (c-b)\{a^2 - (b+c)a + bc\} \\
&= (c-b)(a-b)(a-c) \\
&= (a-b)(b-c)(c-a)
\end{aligned}$$

$$\begin{aligned}
(6) \quad &abx^2 - (a^2+b^2)x + (a^2-b^2) \\
&= abx^2 - (a^2+b^2)x + (a+b)(a-b) \\
&= \{ax - (a+b)\}\{bx - (a-b)\} \\
&= (ax-a-b)(bx-a+b)
\end{aligned}$$

$$\begin{array}{l}
a \quad \times \quad -(a+b) \longrightarrow -ab - b^2 \\
b \quad \times \quad -(a-b) \longrightarrow \frac{-a^2 + ab}{-a^2 - b^2}
\end{array}$$

$$\begin{aligned}
5 \quad (1) \quad &(1+\sqrt{2}+\sqrt{3})^2 \\
&= 1^2 + (\sqrt{2})^2 + (\sqrt{3})^2 + 2 \cdot 1 \cdot \sqrt{2} \\
&\quad + 2 \cdot \sqrt{2} \cdot \sqrt{3} + 2 \cdot \sqrt{3} \cdot 1 \\
&= 1 + 2 + 3 + 2\sqrt{2} + 2\sqrt{6} + 2\sqrt{3} \\
&= 6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}
\end{aligned}$$

$$\begin{aligned}
(2) \quad &(\sqrt{2}+2\sqrt{3}+\sqrt{10})(\sqrt{2}-2\sqrt{3}+\sqrt{10}) \\
&= \{(\sqrt{2}+\sqrt{10})+2\sqrt{3}\}\{(\sqrt{2}+\sqrt{10})-2\sqrt{3}\} \\
&= (\sqrt{2}+\sqrt{10})^2 - (2\sqrt{3})^2 \\
&= (\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot \sqrt{10} + (\sqrt{10})^2 - 12 \\
&= 2 + 2\sqrt{2^2 \times 5} + 10 - 12 = 4\sqrt{5}
\end{aligned}$$

$$\begin{aligned}
(3) \quad &\frac{1}{(\sqrt{3}+1)^2} + \frac{1}{(\sqrt{3}-1)^2} \\
&= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{\{(\sqrt{3}+1)(\sqrt{3}-1)\}^2} \\
&= \frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{(3-1)^2} \\
&= \frac{8}{4} = 2
\end{aligned}$$

$$\begin{aligned}
(4) \quad &\frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} \\
&= \frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2} = \sqrt{2}-1 \\
\frac{1}{\sqrt{3}+\sqrt{2}} &= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\
&= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} \\
&= \sqrt{3}-\sqrt{2} \\
\frac{1}{2+\sqrt{3}} &= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} \\
&= \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = 2-\sqrt{3}
\end{aligned}$$

であるから

$$\begin{aligned}
&\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{2+\sqrt{3}} \\
&= (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (2-\sqrt{3}) \\
&= -1+2 = 1
\end{aligned}$$

教科書 P.47

$$\begin{aligned}
6 \quad (1) \quad &(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3}) \\
&= \{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\} \\
&= (1+\sqrt{2})^2 - (\sqrt{3})^2 \\
&= 1^2 + 2 \times 1 \times \sqrt{2} + (\sqrt{2})^2 - (\sqrt{3})^2 \\
&= 1 + 2\sqrt{2} + 2 - 3 = 2\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
(2) \quad &(1) \text{ より} \\
&\frac{1}{1+\sqrt{2}+\sqrt{3}} \\
&= \frac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3})}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{1 + \sqrt{2} - \sqrt{3}}{2\sqrt{2}} \\
 &= \frac{(1 + \sqrt{2} - \sqrt{3})\sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\
 &= \frac{2 + \sqrt{2} - \sqrt{6}}{4}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad x &= \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{(\sqrt{5} - \sqrt{3})^2}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} \\
 &= \frac{(\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2} \\
 &= \frac{5 - 2\sqrt{15} + 3}{5 - 3} = \frac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15} \\
 \frac{1}{x} &= \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{(\sqrt{5} + \sqrt{3})^2}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \\
 &= \frac{(\sqrt{5})^2 + 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2} \\
 &= \frac{5 + 2\sqrt{15} + 3}{5 - 3} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}
 \end{aligned}$$

$$(1) \quad x + \frac{1}{x} = (4 - \sqrt{15}) + (4 + \sqrt{15}) = 8$$

$$\begin{aligned}
 (2) \quad x^2 + \frac{1}{x^2} &= \left(x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}\right) - 2 \cdot x \cdot \frac{1}{x} \\
 &= \left(x + \frac{1}{x}\right)^2 - 2 = 8^2 - 2 = 62
 \end{aligned}$$

$$\begin{aligned}
 8 \quad &\sqrt{x^2 - 2x + 1} + \sqrt{x^2 + 2x + 1} \\
 &= \sqrt{(x-1)^2} + \sqrt{(x+1)^2} = |x-1| + |x+1|
 \end{aligned}$$

(1) $x < -1$ の場合

$$\begin{aligned}
 x-1 &< -2 < 0, \quad x+1 < 0 \text{ であるから} \\
 &\sqrt{x^2 - 2x + 1} + \sqrt{x^2 + 2x + 1} \\
 &= -(x-1) - (x+1) = -2x
 \end{aligned}$$

(2) $-1 \leq x \leq 1$ の場合

$$\begin{aligned}
 x-1 &\leq 0, \quad 0 \leq x+1 \text{ であるから} \\
 &\sqrt{x^2 - 2x + 1} + \sqrt{x^2 + 2x + 1} \\
 &= -(x-1) + (x+1) = 2
 \end{aligned}$$

(3) $1 < x$ の場合

$$\begin{aligned}
 0 &< x-1, \quad 0 < 2 < x+1 \text{ であるから} \\
 &\sqrt{x^2 + 2x + 1} + \sqrt{x^2 + 2x + 1} \\
 &= (x-1) + (x+1) = 2x
 \end{aligned}$$

9 りんごの個数を x 個とすると、かきの個数は $(20-x)$ 個である。

重さと代金についての次の連立不等式

$$\begin{cases} 220x + 140(20-x) \geq 3700 & \cdots \cdots \textcircled{1} \\ 160x + 80(20-x) \leq 2600 & \cdots \cdots \textcircled{2} \end{cases}$$

を解けばよい。

①の両辺を10で割って

$$\begin{aligned}
 22x + 14(20-x) &\geq 370 \\
 22x + 280 - 14x &\geq 370
 \end{aligned}$$

$$8x \geq 90$$

$$x \geq 11.25 \quad \cdots \cdots \textcircled{3}$$

②の両辺を10で割って

$$\begin{aligned}
 16x + 8(20-x) &\leq 260 \\
 16x + 160 - 8x &\leq 260
 \end{aligned}$$

$$8x \leq 100$$

$$x \leq 12.5 \quad \cdots \cdots \textcircled{4}$$

③, ④より $11.25 \leq x \leq 12.5$

これを満たす整数 x は $x = 12$

$$20 - x = 20 - 12 = 8$$

したがって、りんご 12個、かき 8個

10 (1) $2.5 \leq a < 3.5$

(2) $4.5 \leq b < 5.5$

(3) (1), (2)より $7 \leq a + b < 9$

(4) (1)より $5 \leq 2a < 7$

(2)より $-16.5 < -3b \leq -13.5$

よって $-11.5 < 2a - 3b < -6.5$

11 $-x + 8 \leq 3x \leq x + a$ より

$$\begin{cases} -x + 8 \leq 3x & \cdots \cdots \textcircled{1} \\ 3x \leq x + a & \cdots \cdots \textcircled{2} \end{cases}$$

①より

$$-x - 3x \leq -8$$

$$-4x \leq -8$$

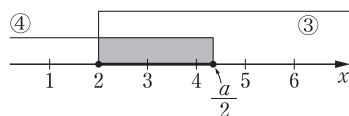
$$x \geq 2 \quad \cdots \cdots \textcircled{3}$$

②より

$$3x - x \leq a$$

$$2x \leq a$$

$$x \leq \frac{a}{2} \quad \cdots \cdots \textcircled{4}$$



$4 \leq \frac{a}{2} < 5$ のとき、整数 x は 2, 3, 4 のちよ

うど 3 個になる。

よって $8 \leq a < 10$